Lect. 2: Light as EM Waves

In the beginning, God said

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \bullet \overline{D} = \rho \qquad \begin{cases} \overline{D} = \varepsilon \overline{E} \\ \overline{B} = \mu \overline{H} \end{cases}$$

$$\nabla \bullet \overline{B} = 0$$

Then, there was light!

$$\text{Material Parameters} \begin{cases} \varepsilon : \text{permittivity} \\ \mu : \text{permeability} \end{cases}$$

Wave Equations

(source free, uniform medium)

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

Light is EM waves!

Solutions for Wave Equations

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

(Plane-wave solutions)

$$\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)} \quad \text{and} \quad \overline{H} = \overline{y} H_0 e^{j(\omega t - kz)}$$

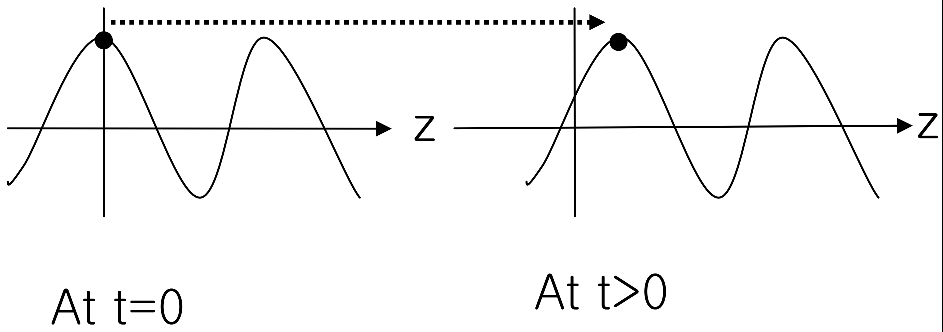
$$\omega = 2\pi f \qquad k = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon} \qquad \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \quad [\Omega]$$

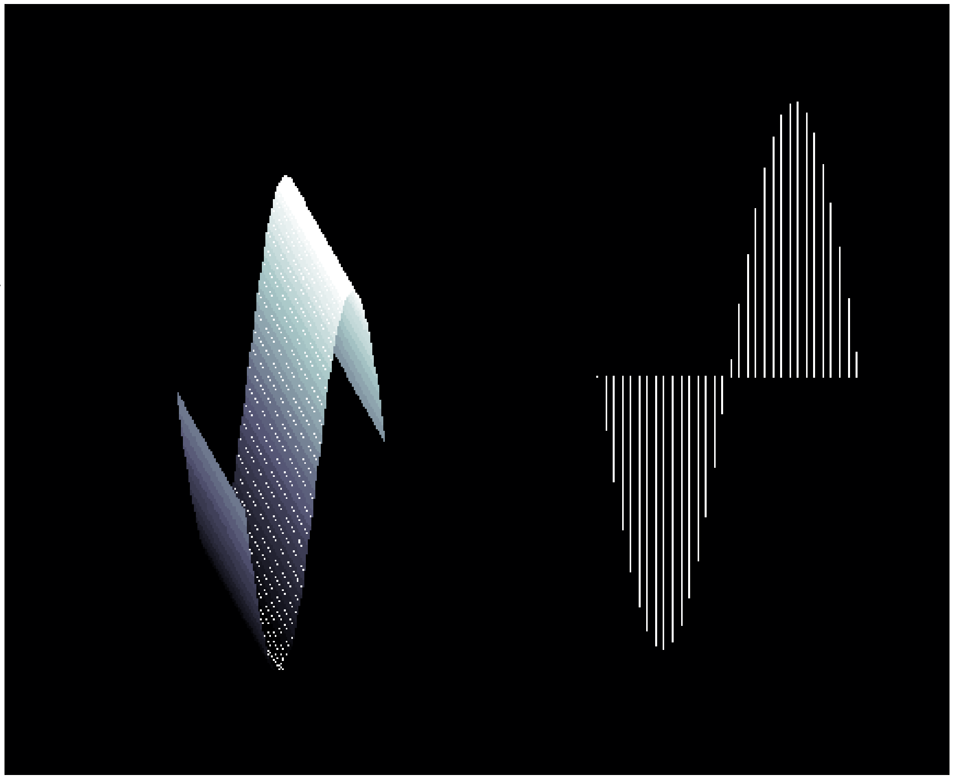
Direction of E, H fields?
Direction of propagation?
Speed of propagation?

$$f \cdot \lambda = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

How does the plane-wave solution look like?

For physical representation, $\mathrm{Re}\left[\bar{x}E_0 e^{j(\omega t - kz)}\right] = \bar{x}E_0 \cos(\omega t - kz)$





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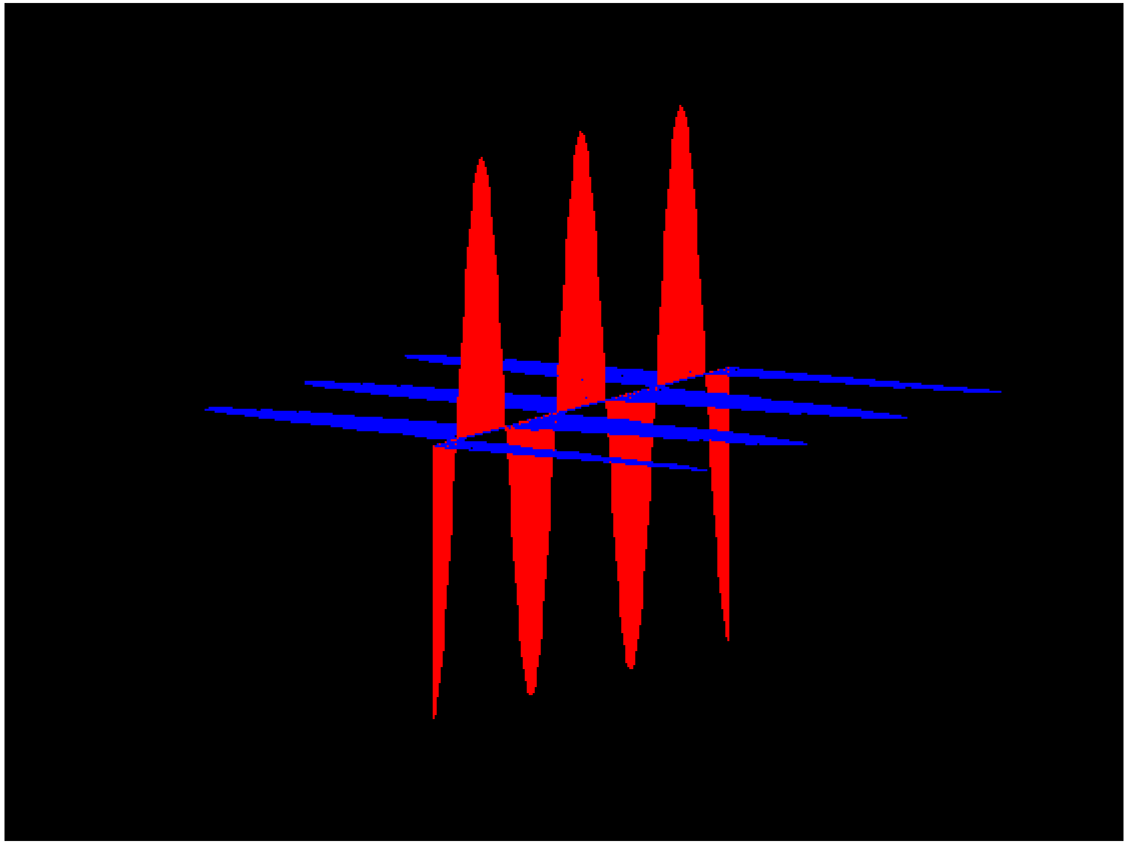
How does the plane-wave solution look like?



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Propagation Directions

+z direction: $e^{j(\omega t - kz)}$ or $e^{-j(\omega t - kz)}$

-z direction: $e^{j(\omega t + kz)}$ or $e^{-j(\omega t + kz)}$

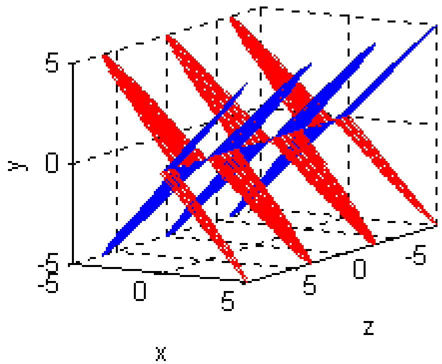
+y direction: $e^{j(\omega t - ky)}$ or $e^{-j(\omega t - ky)}$

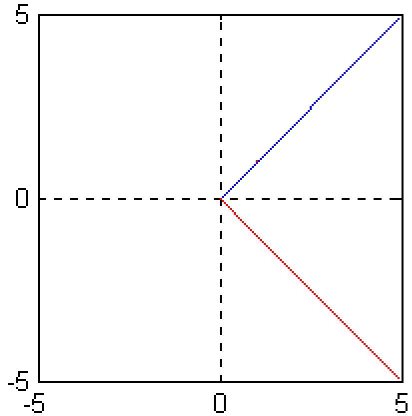
Arbitarary direction, $\overline{k} = \overline{x}k_x + \overline{y}k_y + \overline{z}k_z$ with $\overline{R} = \overline{x}x + \overline{y}y + \overline{z}z$

$$e^{j(\omega t - \overline{k} \bullet \overline{R})} = e^{j\omega t} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

Polarization: Change of E-field direction with time

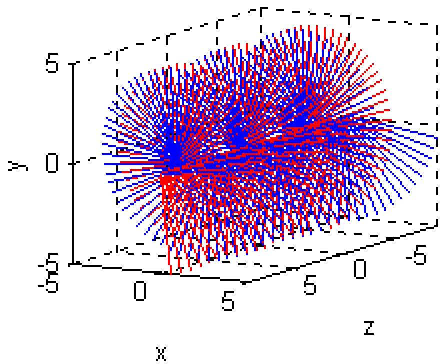
Linear Polarization $\overline{E} = \left(\overline{x}E_0 + \overline{y}E_0\right)e^{j\omega t}e^{jkz}$

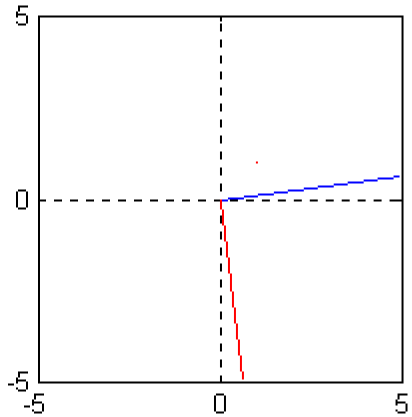




Circular Polarization

$$\overline{E} = \left(\overline{x}E_0 + \overline{y}jE_0\right)e^{j\omega t}e^{jkz}$$





Handedness?

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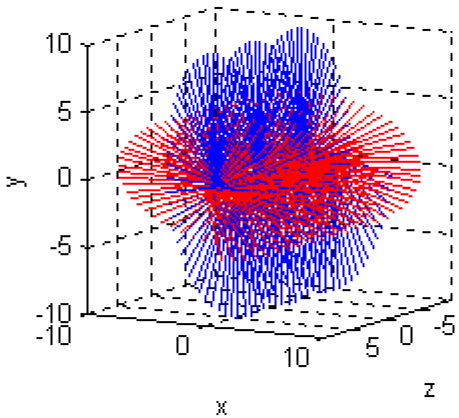
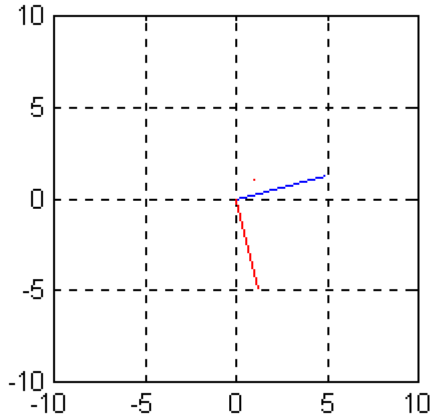
Elliptical Polarization

$$\overline{E} = \left(\overline{x}E_0 + \overline{y}j2E_0\right)e^{j\omega t}e^{jkz}$$





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Characteristics of media: $\begin{cases} \varepsilon : \text{permittivity} \\ \mu : \text{permeability} \end{cases}$ $\begin{cases} \overline{D} = \varepsilon \overline{E} \\ \overline{B} = \mu \overline{H} \end{cases}$

- Assume $\mu = \mu_0$ in this course.
- With different ε (dielectric media), how do plain-wave solutions chnage?

For example, $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$

$$k = \omega\sqrt{\mu\varepsilon} = nk_0 \ (k_0 = \omega\sqrt{\mu\varepsilon_0},\ n = \sqrt{\varepsilon / \varepsilon_0};\ \text{refractive index})$$

$$\Rightarrow \text{change in } \lambda,\ \text{phase velocisty } (v_p = \frac{\omega}{k}),\ \text{group velocity } (v_g = \frac{\partial \omega}{\partial k})$$

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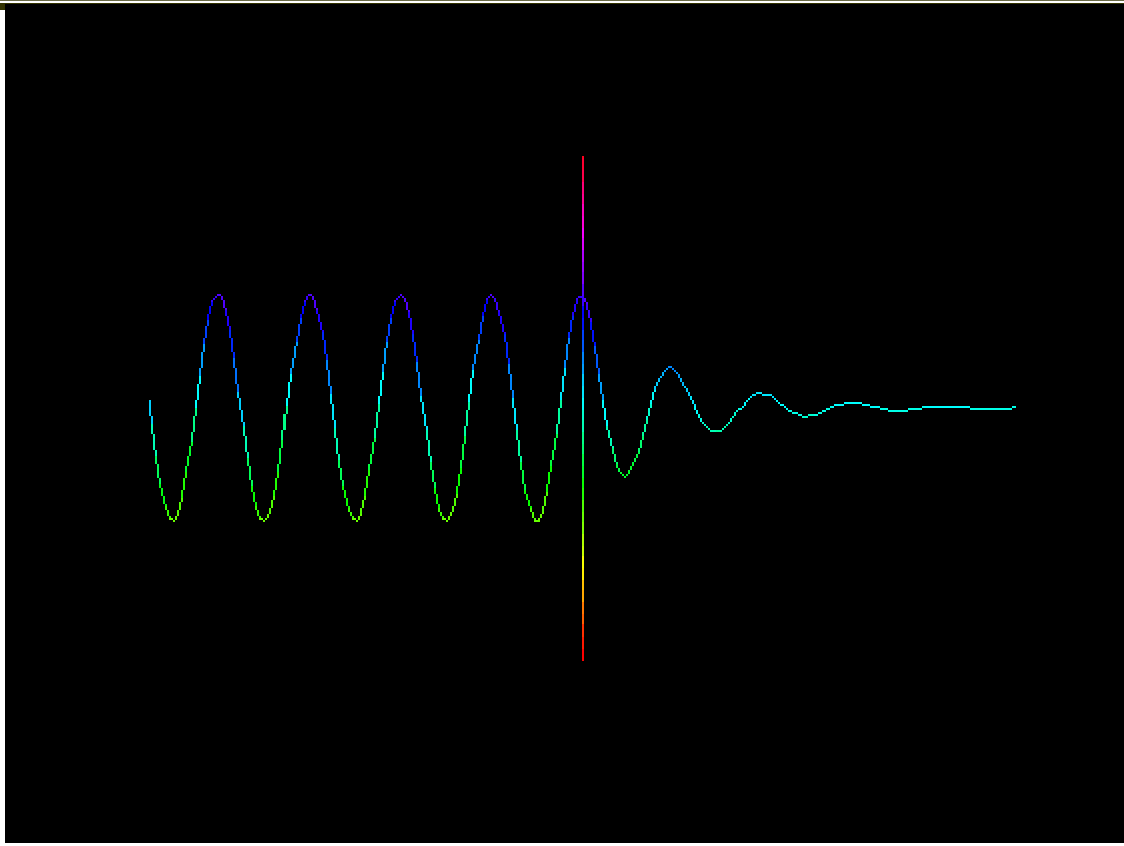
Lossy media

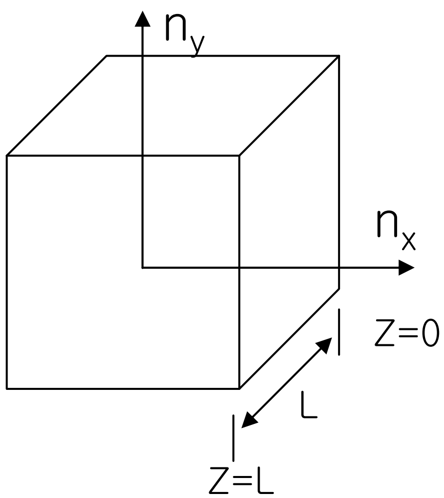


$$\overline{E} = \overline{x}E_0 e^{-j\beta z} e^{-\alpha z} = \overline{x}E_0 e^{-j(\beta - j\alpha)z};\ \text{complex } k$$

(With σ, $\varepsilon_c \equiv \varepsilon - j\dfrac{\sigma}{\omega}$ and $k = \omega\sqrt{\mu\varepsilon_c}$)

– Birefringent Media: Change in polarization

Consider $\vec{E}_{in} = (\bar{x} + \bar{y})E_0 e^{j(\omega t - k_0 z)}$ Incident on a birefringent material (different refractive indices for different directions)



$$\bar{E}_{\text{out}}(z = L) = \bar{x}E_0 e^{-jn_x k_0 L} + \bar{y}E_0 e^{-jn_y k_0 L}$$

$$\frac{E_y}{E_x} = e^{-j(n_x - n_y)k_0 L}$$

$(n_x - n_y)k_0 L = m\pi$: linear polarization

$= \frac{(2m+1)}{2}\pi$: circular polarization

otherwise: elliptical polarization